Ahmed Mahmoud o/ Med salem
/ 7D4 /
Elmaarif

Correction d'une série d'exercices

Exo 1 :

$P(z) = z^3 + 2(\sqrt{2}-1)z^2 + 4(1-\sqrt{2})z - 8$

1) a)

$P(2) = 2^3 + 2(\sqrt{2}-1)2^2 + 4(1-\sqrt{2})2 - 8$

$= 8 + 8\sqrt{2} - 8 + 8 - 8\sqrt{2} - 8$

$\Rightarrow P(2) = 0$

b)

$P(z) = (z-2)(z^2 + az + b)$

tableau d'horner

	1	$2\sqrt{2}-2$	$4-4\sqrt{2}$	-8
2	↓	2	$4\sqrt{2}$	8
	1	$2\sqrt{2}$	4	0
		a	b	

$\Rightarrow a = 2\sqrt{2}$ et $b = 4$

$\Rightarrow P(z) = (z-2)(z^2 + 2\sqrt{2}z + 4)$

2) $P(z) = 0$

On a :

$z - 2 = 0$ ou $z^2 + 2\sqrt{2}z + 4 = 0$

$z = 2$ $\quad \Delta = (2\sqrt{2})^2 - 4(4)(1)$

$\Delta = -8 = (2i\sqrt{2})^2$

$z_1 = \frac{-2\sqrt{2} + 2i\sqrt{2}}{2} = -\sqrt{2} + i\sqrt{2}$

$z_2 = \frac{-2\sqrt{2} - 2i\sqrt{2}}{2} = -\sqrt{2} - i\sqrt{2}$

- vérification :

$z_1 + z_2 = -\sqrt{2} + i\sqrt{2} - \sqrt{2} - i\sqrt{2}$

$\Rightarrow \boxed{z_1 + z_2 = -2\sqrt{2}}$

- module et argument de z_1

$|z_1| = \sqrt{(-\sqrt{2})^2 + (\sqrt{2})^2}$

$= \sqrt{4} = 2$ $\quad \boxed{|z_1| = 2}$

$\cos\theta = -\frac{\sqrt{2}}{2}$

$\sin\theta = \frac{\sqrt{2}}{2}$ $\Rightarrow \theta = \pi - \frac{\pi}{4} \Rightarrow \theta$

- module et argument de z_2

$|z_2| = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2}$

$= \sqrt{4} = 2$ $\quad \boxed{|z_2| = 2}$

$\cos\theta = -\frac{\sqrt{2}}{2}$

$\sin\theta = -\frac{\sqrt{2}}{2}$ $\Rightarrow \theta = \pi + \frac{\pi}{4} \Rightarrow \theta$

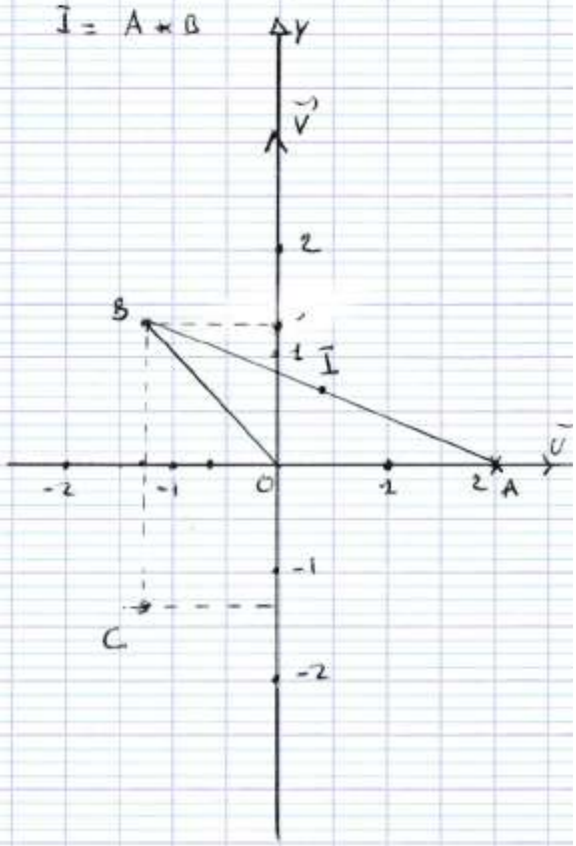
3) a)

$A(2, 0)$

$B = z_1 \Rightarrow B(-\sqrt{2}, \sqrt{2})$

$C = z_2 \Rightarrow C(-\sqrt{2}, -\sqrt{2})$

$I = A * B$



b) $\frac{z_A - z_0}{z_B - z_0} = \frac{z_A}{z_B}$

$\frac{z_A}{z_B} = \frac{2}{-\sqrt{2}+i\sqrt{2}} \times \frac{(-\sqrt{2}-i\sqrt{2})}{(-\sqrt{2}-i\sqrt{2})}$

$= \frac{-2\sqrt{2}-2i\sqrt{2}}{4} = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

$Z = \frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i$

$|Z| = \sqrt{\left(-\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2}$

$= \sqrt{1} = 1$

On a : $|Z| = \left|\frac{z_A - z_0}{z_B - z_0}\right| = 1$

$\Rightarrow$ OAB est isocèle direct en O

On a : $\cos\theta = \frac{a}{|Z|} = \frac{\frac{-1}{\sqrt{2}}}{1} = -\frac{\sqrt{2}}{2}$

$\sin\theta = \frac{b}{|Z|} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\Rightarrow \theta = \frac{5\pi}{4} \Rightarrow (\vec{OB}, \vec{OA}) = \frac{5\pi}{4}$

On a :

$(\vec{OB}, \vec{OA}) = (\vec{OB}, \vec{OI}) + (\vec{OI}, \vec{OA}) = \frac{5\pi}{4}$

$2(\vec{OI}, \vec{OA}) = \frac{5\pi}{4}$

$\Rightarrow (\vec{OI}, \vec{OA}) = (\vec{OI}, \vec{u}) = \frac{5\pi}{8}$

$\Rightarrow \boxed{(\vec{OI}, \vec{u}) = \frac{5\pi}{8}}$

c) $z_1 = \frac{z_A + z_B}{2} = \frac{2-\sqrt{2}+i\sqrt{2}}{2}$

$= \frac{2-\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$ $\quad I\left(\frac{2-\sqrt{2}}{2}; \frac{\sqrt{2}}{2}\right)$

$|z_1| = \sqrt{\frac{(2-\sqrt{2})^2}{4} + \frac{(\sqrt{2})^2}{4}} = \sqrt{\frac{1}{2}+\frac{1}{2}} = 1$

$\Rightarrow \boxed{|z_1| = 1}$

d) On a :

$\cos\frac{3\pi}{8} = \frac{2-\sqrt{2}}{2}$

$\sin\frac{3\pi}{8} = \frac{\sqrt{2}}{2}$

Exo 2 :

$U_0 = 0$, $U_1 = 1$ et $U_{n+2} = 5U$

1) - $U_2 = 5U_1 - 4U_0$

$\boxed{U_2 = 5}$

- $U_3 = 5U_2 - 4U_1$

$= 5(5) - 4(1)$

$\boxed{U_3 = 21}$

- $U_4 = 5U_3 - 4(U_1)$

$= 5(21) - 4(5)$

$\boxed{U_4 = 85}$

2) a)

$U_{n+1} = 4U_n + 1$

$U_1 = 4U_0 + 1 = 1$ vraie

Mq $U_{n+2} = 4U_{n+1} + 1$

$U_{n+1} = 4U_n + 1$

$U_{n+2} = 16U_n + 4 + 1$

$U_{n+2} = 4(4U_n + 1) + 1$

$U_{n+2} = 4U_{n+1} + 1$

$\Rightarrow \boxed{U_{n+1} = 4U_n + 1}$

b) $U_2 = 4U_1 + 1 = 5 \Rightarrow \boxed{U_2 =}$

$U_3 = 4U_2 + 1 = 21 \Rightarrow \boxed{U_3 =}$

3) $V_n = U_n + \frac{1}{3}$

a) $V_{n+1} = 4U_n + 1 + \frac{1}{3} = 4U_n$

$= 4\left(U_n + \frac{1}{3}\right)$

$V_{n+1} = 4V_n$

$\Rightarrow V_n$ est S.G de raison

Corrigé par ahmed mahmoud o/ med salem

$V_0 = U_0 + \frac{1}{3} = \frac{1}{3}$

b) $\boxed{V_n = \frac{1}{3} \times (4)^n}$

$\boxed{U_n = \frac{1}{3} \times (4)^n - \frac{1}{3}}$

c) $U_4 = 1/3 \times (4)^4 - 1/3$

$= 1/3 \times 256 - 1/3 = 85$

$\Rightarrow \boxed{U_4 = 85}$

$U_3 = 1/3 \times (4)^3 - 1/3 = 21$

$\Rightarrow \boxed{U_3 = 21}$

$U_2 = 1/3 \times (4)^2 - 1/3 = 5$

$\Rightarrow \boxed{U_2 = 5}$

4) a)

$S_n = V_0 + V_1 + \dots + V_n$

Comme (V_n) et une S.G

$\Rightarrow S_n = V_0 \frac{1 - (4)^{n+1}}{1-4}$

$= 1/3 \times \frac{1 - (4)^{n+1}}{-3}$

$\boxed{S_n = \frac{(4)^{n+1} - 1}{9}}$

b) $S'_n = U_0 + U_1 + \dots + U_n$

$= (V_0 - \frac{1}{3}) + (V_1 - \frac{1}{3}) + \dots + (V_n - \frac{1}{3})$

$= (V_0 + V_1 + V_2 + \dots + V_n) - \frac{1}{3}n$

$S'_n = S_n - 1/3 n$

$\boxed{S'_n = \frac{(4)^{n+1} - 1}{9} - 1/3 n}$

Fin.