N=1003 Bac 2012 S C

1ème Ecriture

Ex 2 : $f(x) = \frac{1}{x\ln x}$

$D_f =]0,1[\cup]1,+\infty[$

1-a) * $\lim_{0^+} f(x) = \frac{1}{0^-} = -\infty$

$\boxed{\lim_{0^+} f(x) = -\infty}$ $x=0$ A.V

* $\lim_{1^-} f(x) = \frac{1}{0^-} = -\infty$

$\boxed{\lim_{1^-} f(x) = -\infty}$ $x=1$ A.V

* $\lim_{1^+} f(x) = \frac{1}{0^+} = +\infty$ $\boxed{\lim_{1^+} f(x) = +\infty}$

$x=1$ A.V

* $\lim_{+\infty} f(x) = \frac{1}{+\infty} = 0$ $y=0$ A.H

$\boxed{\lim_{+\infty} f(x) = 0}$

2) T.V de f

~~$f'(x) = \frac{\ln x}{(x\ln x)^2}$ 2ème~~

~~le signe de f'(x) est celui de~~

~~$x\ln x$ dont $f'(x)=0$~~

~~$x\ln x = 0 \Rightarrow \ln x = 0 \Rightarrow x = e^0 = 1$~~

$f'(x) = \frac{0 \times \dots - 1 \times (\ln x + 1)}{(x\ln x)^2}$

$f'(x) = \frac{-(1+\ln x)}{(x\ln x)^2}$ 2ème le signe

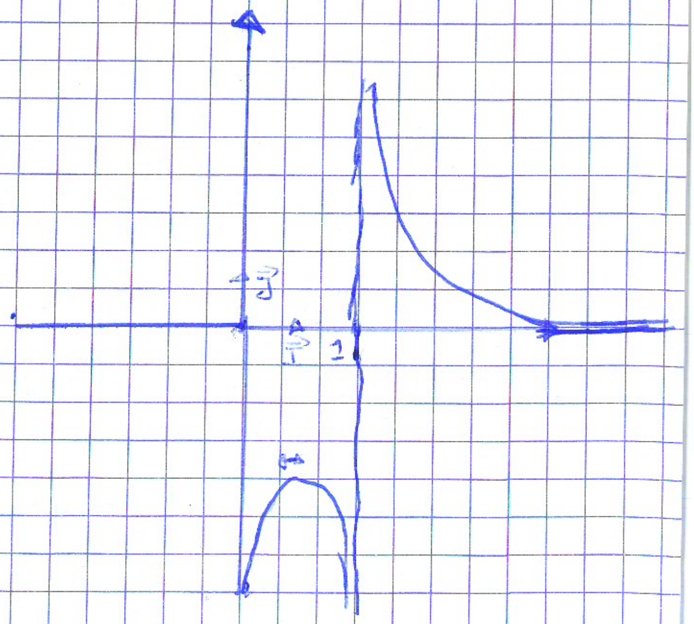
de $f'(x)$ est celui de $(1+\ln x)=0$

$1+\ln x = 0$, $\ln x = -1$, $x = e^{-1}$, $x = \frac{1}{e}$

$f(\frac{1}{e}) = -e$ on dresse le T.V

x	0		$\frac{1}{e}$		1		$+\infty$
$f'(x)$	||	$+$	0	$-$	||	$-$	
$f(x)$	|| $-\infty$	↗	$-e$	↘	$-\infty$ || $+\infty$	↘	0

c) la courbe C



1007 Bac 2012 5-C

Écriture pour Ex 02

1) $f(x) = \dfrac{1}{x\ln x}$ $\quad D_f =]0;1[\cup]1;+\infty[$

* limite : * $\lim\limits_{0^+} f(x) = \dfrac{1}{0^-} = -\infty$

$\boxed{\lim f(x) = -\infty}$ $\quad x = 0$ A V

$\lim\limits_{1^-} f(x) = \dfrac{1}{0^-} \Rightarrow \boxed{\lim\limits_{1^-} f(x) = -\infty}$

* $\lim\limits_{1^+} f(x) = \dfrac{1}{0^+} \Rightarrow \boxed{\lim\limits_{1^+} f(x) = +\infty}$

$x = 1$ A V

* $\lim\limits_{+\infty} f(x) = \dfrac{1}{+\infty \times +\infty} \Rightarrow \lim\limits_{+\infty} f(x) = \dfrac{1}{+\infty}$

$\boxed{\lim\limits_{+\infty} f(x) = 0}$ $\quad y = 0$: A.H

b) T.V de f

$f'(x) = \dfrac{0 \times x\ln x - 1 \times (\ln x + 1)}{(x\ln x)^2}$

$f'(x) = \dfrac{-(1+\ln x)}{(x\ln x)^2}$ donc le signe de $f'(x)$ est celui de $(1+\ln x)$

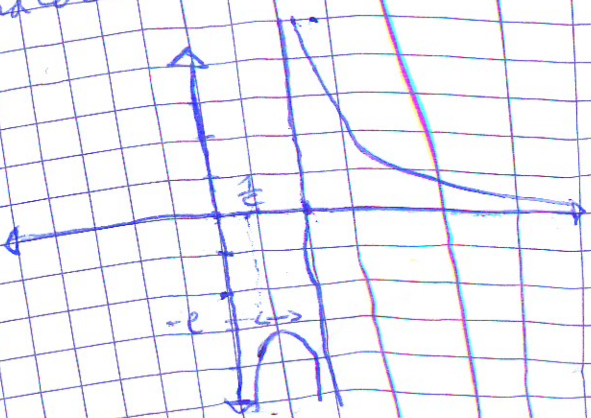
donc $f'(x) = 0 \Rightarrow 1 + \ln x = 0 \Rightarrow \ln x = -1$

$x = e^{-1} \Rightarrow \boxed{x = \frac{1}{e}} \Rightarrow f\left(\frac{1}{e}\right) = \dfrac{1}{\frac{1}{e} \times -1}$

$\Rightarrow f\left(\frac{1}{e}\right) = -e$

x	0		$\frac{1}{e}$		1		$+\infty$
$f'(x)$		$+$	0	$-$			
$f(x)$	$-\infty$	↗	$-e$	↘ $-\infty$		$+\infty$ ↘	0

c) Tracer la courbe



2) $\forall n \geq 2$

$U_n = \sum\limits_{k=2}^{n} \dfrac{1}{k\ln k} - \ln(\ln n) = \dfrac{1}{2\ln 2} + \dfrac{1}{3\ln 3} + \dots$

a) Montrer que $\forall n \geq 2$

$\dfrac{1}{(n+2)\ln(n+2)} \leq \int_n^{n+2} f(t)\,dt \leq \dfrac{1}{n\ln n}$

$n \leq t \leq n+2 \Rightarrow \ln(n) \leq \ln t \leq \dots$

$\dfrac{1}{(n+1)\ln(n+2)} \leq f(t) \leq \dfrac{1}{n\ln n}$

$\int_n^{n+2} \dfrac{dt}{(n+1)\ln(n+2)} \leq \int_n^{n+2} f(t)\,dt \leq \int_n^{n+2} \dfrac{dt}{n\ln n}$

$\dfrac{1}{(n+1)\ln(n+2)} \leq \int_n^{n+2} f(t)\,dt \leq \dfrac{1}{n\ln n}$

N° 1003 BAC 2012 SC Exo 2

3ème Écriture

Exo 2 : $f(x) = \frac{1}{x\ln x}$ $D_f\ ]0,1[\ \cup\ ]1,+\infty[$

1-a) les limites

* $\lim_{0^+} f(x) = \frac{1}{0^-} \Rightarrow \boxed{\lim_{0^+} f(x) = -\infty}$

$x = 0$ A.V au voisinage $-\infty$

* $\lim_{1^-} f(x) = \frac{1}{0^-} \Rightarrow \boxed{\lim_{1^-} f(x) = -\infty}$

* $\lim_{1^+} f(x) = \frac{1}{0^+} \Rightarrow \boxed{\lim_{1^+} f(x) = +\infty}$

$x = 1$ A.V au voisinage $+\infty$

* $\lim_{+\infty} f(x) = \frac{1}{+\infty \times +\infty} \Rightarrow \lim_{+\infty} f(x) = \frac{1}{+\infty} = 0$

$\boxed{\lim_{+\infty} f(x) = 0}$ $y = 0$ A.H au voisinage $+\infty$

b) T.V de f : $f'(x) = \frac{0 \times x\ln x - 1(\ln x + 1)}{(x\ln x)^2}$

$f'(x) = \frac{-(\ln x + 1)}{(x\ln x)^2}$ le signe de $f'(x)$ et celui de $(\ln x + 1)$

$f'(x) = 0 \Rightarrow \ln x + 1 = 0 \Rightarrow \ln x = -1$
$\Rightarrow x = e^{-1} \Rightarrow \boxed{x = \frac{1}{e}}$

$f(\frac{1}{e}) = \frac{1}{\frac{1}{e}\ln\frac{1}{e}} = -e$

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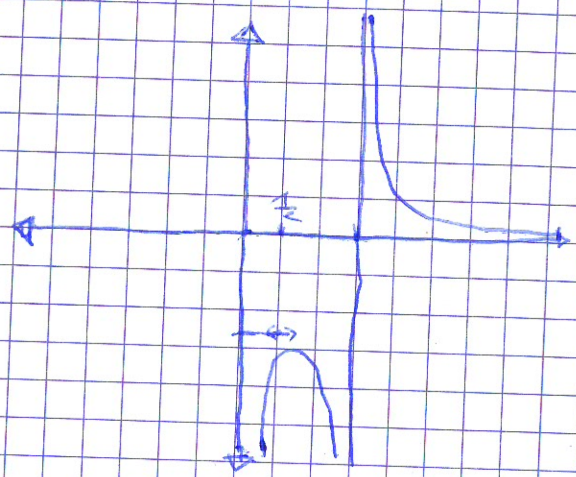
x	0		$\frac{1}{e}$		1		
$f'(x)$		$+$	0	$-$		$-$	
$f(x)$		$-\infty \nearrow$	$-e$	$\searrow -\infty$		$+\infty \searrow$	0

c) Construire la courbe C



2) $\forall n \geq 2$ $U_n = \sum_{k=0}^{n} \frac{1}{k\ln k} - \ln(\ln n) = \frac{1}{2\ln 2} + \frac{1}{3\ln 3} + \dots + \frac{1}{n\ln n} - \ln(\ln n)$

a) Montrons que $\forall n \geq 2$

$\frac{1}{(n+1)\ln(n+1)} \leq \int_n^{n+1} f(t)\,dt \leq \frac{1}{n\ln n}$

$n \leq t \leq n+1 \Rightarrow \ln n \leq \ln t \leq \ln(n+1)$

$\frac{1}{(n+1)\ln(n+1)} \leq f(t) \leq \frac{1}{n\ln n}$

$\int_n^{n+1} \frac{dt}{(n+1)\ln(n+1)} \leq \int_n^{n+1} f(t)\,dt \leq \int_n^{n+1} \frac{dt}{n\ln n}$

$\frac{1}{(n+1)\ln(n+1)} \leq \int_n^{n+1} f(t)\,dt \leq \frac{1}{n\ln n}$