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Classe: 7C

Exercice 1:

Déterminer le module et un argument, puis écrire sous forme algebrique chacun des nombre complexes suivant:

$(2-2i)^5$; $\frac{1}{(1-i\sqrt{3})^{10}}$; $\frac{1+i}{1-i\sqrt{3}}$; $\frac{1+\sqrt{2}+i}{1+\sqrt{2}-i}$, $\left(\sqrt{2+\sqrt{2}}+i\sqrt{2-\sqrt{2}}\right)^8$, $\frac{(-2i)(1+i\sqrt{3})^{12}}{(1+i)^4}$

Solution:

$* z_1 = (2-2i)^5 = \left(2\sqrt{2}e^{-i\frac{\pi}{4}}\right)^5$

$= (2\sqrt{2})^5 e^{-i\frac{5\pi}{4}} = 2^7\sqrt{2}e^{i\frac{3\pi}{4}}$

$|z_1| = 2^7\sqrt{2}$, $\arg z_1 = \frac{3\pi}{4}$

$z_1 = 2^7\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$= 2^7\sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$

$2^7(-1+i)$

$\boxed{z_1 = -2^7 + 2^7 i}$ Forme algebrique

$* z_2 = \frac{1}{1-i\sqrt{3}} = (1-i\sqrt{3})^{-10}$

$= \left(2e^{-i\frac{\pi}{3}}\right)^{-10} = 2^{-10}e^{i\frac{10\pi}{3}}$

$z_2 = 2^{-10}e^{\frac{\pi}{3}} \Rightarrow \boxed{|z_2| = 2^{-10} ; \arg z = \frac{-\pi}{3}}$

$z_2 = 2^{-10}\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$

$z_2 = 2^{-10}\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$z_2 = 2^{-11}(1-\sqrt{3})$

$\boxed{z_2 = 2^{-11} - 2^{-11}i\sqrt{3}}$ Forme algebrique

$* z_3 = \frac{1+i}{1-i\sqrt{3}} \Rightarrow |z_3| = \frac{|1+i|}{|1-i\sqrt{3}|} = |z_3| = \frac{\sqrt{2}}{2}$

$\arg z_3 = \arg(1+i) - \arg(1-\sqrt{3})$

$= \frac{\pi}{4} - \left(-\frac{\pi}{3}\right) = \frac{7\pi}{12}$

$z_3 = \frac{(1+i)(1+i\sqrt{3})}{4} = z_3 = \frac{(1+i)(1+i\sqrt{3})}{(1-i\sqrt{3})(1+i\sqrt{3})}$

$z_3 = \frac{1+i\sqrt{3}+i-\sqrt{3}}{4}$

$\boxed{z_3 = \frac{1-\sqrt{3}}{4} + \frac{1+\sqrt{3}i}{4}}$

$* z_4 = \frac{1+\sqrt{2}+i}{1+\sqrt{2}+i}$ du type $\frac{v}{u}$

donc $|z_4| = 1$

$z_4 = \frac{(1+\sqrt{2}+i)(1+\sqrt{2}+i)}{(1+\sqrt{2}-i)(1+\sqrt{2}+i)}$

$z_4 = \frac{(1+\sqrt{2})^2 - 1^2 + 2(1+\sqrt{2})i}{(1+\sqrt{2})^2+1}$

$= \frac{2+2\sqrt{2}+2(1+\sqrt{2})i}{2(2+\sqrt{2})} = z_4 = \frac{1+\sqrt{2}}{2\sqrt{2}}(1+i)$

$\arg z_4 = \frac{\pi}{4}$ Forme Algebrique $\boxed{z_4 = \frac{1+\sqrt{2}}{2+\sqrt{2}} + \frac{1+\sqrt{2}}{2+\sqrt{2}}i}$

$* z_5 = \left(\sqrt{2+\sqrt{2}} + i\sqrt{2-\sqrt{2}}\right)^8$

$z_5 = \left(\left(\sqrt{2+\sqrt{2}} + i\sqrt{2-\sqrt{2}}\right)^2\right)^4$

$= \left(2+\sqrt{2} - 2 + \sqrt{2} + 2i\sqrt{2}\right)^4 = \left(2\sqrt{2} + 2i\sqrt{2}\right)^4$

$\left(2\sqrt{2}(1+i)\right)^4 = \left(2\sqrt{2}\cdot\sqrt{2}e^{i\frac{\pi}{4}}\right)^4 = \left(4e^{i\frac{\pi}{4}}\right)^4$

$\boxed{\Rightarrow |z_5| = 256 . \arg z_5 = \pi}$

$\boxed{z_5 = -256 ; \text{F. Algebrique}}$

$z_6 = \frac{(-2i)(1+i\sqrt{3})^{12}}{(1+i)^4} = z_6 = \frac{(-2i)(1+i\sqrt{3})^{12}}{(1+i)^4}$

$= \frac{2 \cdot 2^{12}}{(\sqrt{2})^4} = \frac{2^{13}}{2^2} = |z_6| = 2^{11}$

$\arg z_6 = \arg(-2i) + 12\arg(1+i\sqrt{3}) - 4\arg(1+i) = \frac{-\pi}{2} + 12\cdot\frac{\pi}{3} - 4\cdot\frac{\pi}{4} = \frac{-3\pi}{2} = +\frac{\pi}{2}\ [2\pi]$

$\arg z_6 = \frac{\pi}{2}$

$z_6 = 2^{11}e^{i\frac{\pi}{2}} = \boxed{z_6 = 2^{11}i}$