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Bac 2013 : S.N

Exercice : 1

1) (E) : $25x - 9y = 5$.

a) $25u + 9v = 1$

$$\begin{array}{r|l} 25 & 9 \\ 18 & 2 \\ \hline 07 & \end{array} \qquad \begin{array}{r|l} 9 & 7 \\ 7 & 1 \\ \hline 2 & \end{array}$$

$$\begin{array}{r|l} 7 & 2 \\ 6 & 3 \\ \hline 1 & \end{array} \qquad \begin{array}{r|l} 2 & 1 \\ 2 & 2 \\ \hline 0 & \end{array}$$

$\Rightarrow 25 \wedge 9 = 1$.

$25 = 9 \times 2 + 7$

$9 = 7 \times 1 + 2$

$7 = 2 \times 3 + 1$

$7 = 25 - 9 \times 2$

$2 = 9 - 7 \times 1$

$1 = 7 - 2 \times 3$

$*\ 1 = 7 - 2 \times 3$

$= 25 - 9 \times 2 - (9 - 7 \times 1) \times 3$

$= 25 - 9 \times 2 - 9 \times 3$

$+ (25 - 9 \times 2) \times 3$

$= 25 - 9 \times 2 - 9 \times 3 + 25 \times 3$

$- 9 \times 2 \times 3$

$= (4) \times 25 + 9 \times (-11)$

$\Rightarrow 25(4) + 9(-11) = 1$.

$\Rightarrow (u, v) = (4, -11)$.

$25(4) + 9(-11) = 1$

$\Rightarrow 25(4)(5) - 9(11)(5) = 5$

$\Rightarrow 25 \times 20 - 9 \times 55 = 5$

$\Rightarrow (x_0, y_0) = (20, 55)$.

est une solution particullière de (E).

b) $25x - 9y = 5$

$\Rightarrow 25x - 9y = 25 \times 20 - 9 \times 55$

$25(x - 20) = 9(y - 55)$

$25 \wedge 9$ et $25 / 9(y - 55)$.

$\Rightarrow 25 / y - 55$.

$\Rightarrow y - 55 = 25K$

$\Rightarrow y = 55 + 25K$

$25 \wedge 9 = 1$ et $9 / 25(x-20)$

$\Rightarrow 9 / x - 20 \Rightarrow$

$x - 20 = 9k \Rightarrow$

$x = 20 + 9k$.

$\Rightarrow$

$\begin{cases} x = 20 + 9k \\ y = 55 + 25k \end{cases}$ $k \in \mathbb{Z}$.

. En remplaçant :

$25(9k+20) - 9(25k+55)$

$= \cancel{25 \times 9k} + 25 \times 20 - \cancel{9 \times 25k}$

$- 9 \times 55 = 1$.

2) PGCD $(x, y) = d$.

a) PGCD $(x, y) = d$

$\Rightarrow ax + by = d$.

$\Rightarrow d / ax + by$.

$d / 25x - 9y$

or $25x - 9y = 5$

$\Rightarrow d/5 \Rightarrow d$ peut être inclu seulement dans les diviseurs positifs de 5.

$\Rightarrow d \in \{1, 5\}$.

b) x et y sont premiers

$\Rightarrow d = 1$.

$\Rightarrow x$ et y ne sont pas divisible par 5.

$\Rightarrow$ les solutions sont :

$\begin{cases} x = 20 + 9k \\ y = 55 + 25k \end{cases}$.

avec $k \in \mathbb{Z}$ et k n'est pas un multiple de 5.

c) $25x^2 - 9y^2 = 5$

$\Rightarrow (5x)^2 - (3y)^2 = 5$

$\Rightarrow (5x - 3y)(5x + 3y) = 5$

$\Rightarrow 5x - 3y / 5$ et $(5x + 3y) / 5$

$\Rightarrow \begin{cases} 5x - 3y = 5 \\ 5x + 3y = 1 \end{cases}$ ou $\begin{cases} 5x - 3y = 1 \\ 5x + 3y = 5 \end{cases}$

$\Rightarrow \begin{cases} -3y - 3y = 1 \\ -6y = 1 \end{cases} \Rightarrow \boxed{y = -1/6}$

ou $\begin{cases} 3y + 3y = 5 \\ 6y = 5 \end{cases} \Rightarrow \boxed{y = 5/6}$

$\Rightarrow$ Non, car $-1/6$ et $5/6 \notin \mathbb{Z}$.

Exercice ②

• $P(z) = z^3 - (9-i)z^2 + (28-5i)z - 32 + 4i$.

1) a)

$P(4) = 4^3 - (9-i)(4)^2 + (28-5i)(4) - 32 + 4i$

$= 64 - 144 + 16i + 112 - 20i - 32 + 4i$

$= 176 + 20i - 176 - 20i$

$= 0$

$\Rightarrow \boxed{P(4) = 0}$

$P(z) = (z-4)(z^2 + az + b)$.

	1	$-9+i$	$28-5i$	$-32+4i$
4	$\downarrow$	4	$-20+4i$	$32-4i$
	1	$-5+i$	$8-i$	0

$\Rightarrow P(z) = (z-4)(z^2 + (-5+i)z + 8 - i)$

b) $P(z) = 0 \Rightarrow$

$z - 4 = 0$ ou $z^2 + (-5+i)z + 8 - i = 0$

$\boxed{z = 4}$

$\Delta = b^2 - 4ac$

$\Delta = (-5+i)^2 - 4(8-i)$

$\Delta = 24 - 10i - 32 + 4i$

$\Delta = -8 - 6i$

$\Rightarrow \sqrt{\Delta} = (3i-1)^2$.

$\Rightarrow z = \dfrac{5-i-3i+1}{2}$ ou $z = \dfrac{5-i+3i-1}{2}$

$\Rightarrow z = 3-2i$ ou $z = 2+i$

$S = \{4 \,;\, 3-2i \,;\, 2+i\}$.

2) $\begin{cases} z_A = 4 \\ z_B = 2+i \\ z_C = 3-2i \end{cases}$

a) $A \xrightarrow{S} B$

$C \xrightarrow{S} C$

$z' = az + b \Rightarrow$

$\begin{cases} z_B = a z_A + b \\ z_C = a z_C + b \end{cases}$

$\begin{cases} b = z_B - a z_A \\ b = z_C - a z_C \end{cases}$

$\Rightarrow z_B - a z_a = z_c - a z_c$.

$a z_c - a z_a = z_c - z_B$

$a = \dfrac{z_c - z_B}{z_c - z_a} = \dfrac{3-2i-2-i}{3-2i-4}$

$\Rightarrow a = \dfrac{1-3i}{-1-2i} = \dfrac{(1-3i)(-1+2i)}{(-1-2i)(-1+2i)}$

$\Rightarrow a = \dfrac{5+5i}{5} \Rightarrow$

$a = 1+i \Rightarrow b = 2+i-4-4i$

$b = -2-3i \Rightarrow$

$\boxed{z' = (1+i)z - 2 - 3i}$

b) le rapport $k = |1+i| = \sqrt{2}$

l'angle $\theta = Arg(1+i) = \frac{\pi}{4}\ [2\pi]$.

3)° a) $\varphi(z) = (x+iy)^2 - (5-i)(x+iy) + 8 - i =$

$x^2 - y^2 + 2xyi - 5x - 5iy + xi - y + 8 - i$

$\Rightarrow \varphi(z) = x^2 - y^2 - 5x - y + 8 + i(2xy - 5y + x - 1)$

$\Gamma = \{ M \in P / \varphi(z)$ est imaginaire pur $\}$.

$\Rightarrow x^2 - y^2 - 5x - y + 8 = 0$

$\Rightarrow (x - \frac{5}{2})^2 - (y + 1/2)^2 - \frac{24}{4} + 8 = 0$

$\Rightarrow (x - 5/2)^2 - (y + 1/2)^2 = 2$

$\Rightarrow -\frac{(x - 5/2)^2}{(\sqrt{2})^2} + \frac{(y + 1/2)^2}{(\sqrt{2})^2} = 1.$

Γ a pour équation :

$$-\frac{X^2}{(\sqrt{2})^2} + \frac{Y^2}{(\sqrt{2})^2} = 1$$

dans le repère $(\Omega, \vec{i}, \vec{j})$ $\Omega(\frac{5}{2}; -1/2)$. D'où Γ est une hyperbole de centre Ω et sommets $B(0, \sqrt{2})$ dans le repère $(\Omega, \vec{i}, \vec{j})$ et $B'(0, -\sqrt{2})$.

• dans le repère $(O, \vec{i}, \vec{j})$ on a $B(\frac{5}{2}, \sqrt{2} - \frac{1}{2})$ et $B'(\frac{5}{2}, -\sqrt{2} - \frac{1}{2})$.

Les asymptotes Δ et Δ' ont pour équations dans le repère $(O, \vec{i}, \vec{j})$ on a : $\cancel{Y} = 1/2$

$\Delta : Y = \frac{\sqrt{2}}{\sqrt{2}} X = X$ et

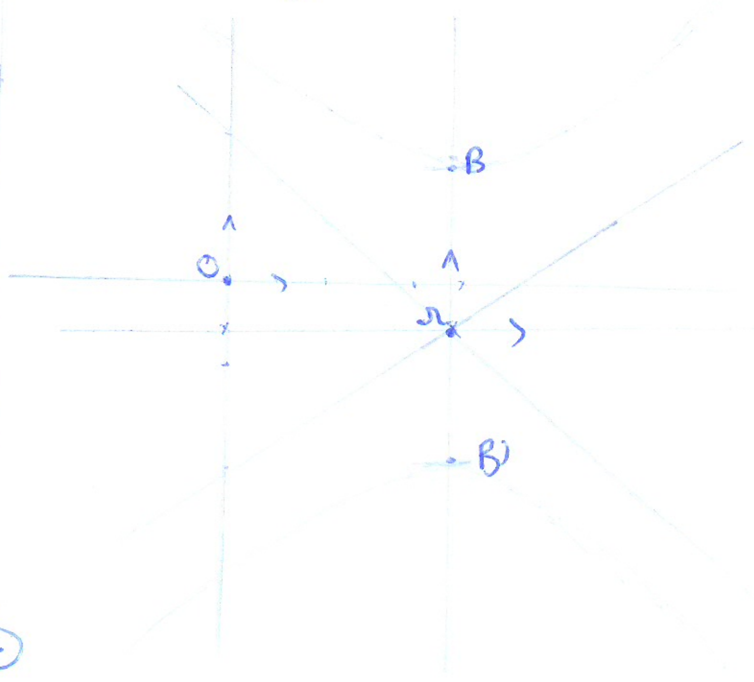
$\Delta' : Y = -\frac{\sqrt{2}}{\sqrt{2}} X = -X$

et dans le repère $(O, \vec{i}, \vec{j})$ on a : $y + 1/2 = x - 5/2$

$\Rightarrow \Delta : y = x - 3$

et $y + \frac{1}{2} = -x + \frac{5}{2}$

$\Rightarrow \Delta' : y = -x + 2$.



Exercice : 3

$f(x) = (3-x)e^x$

1)- a)- • $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} (3-x)e^x$

$= \lim\limits_{x \to -\infty} \cancel{3e^x}^{\,0} - x e^x = -1 \times 0 = 0$

$\Rightarrow \lim\limits_{x \to -\infty} f(x) = 0 \Rightarrow y = 0$ A.H

• $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} (3-x)e^x$

$= \lim\limits_{x \to +\infty} 3e^x - x \dfrac{e^x}{x} =$ F.I.

$= \lim\limits_{x \to +\infty} (3-x) \times \lim\limits_{x \to +\infty} e^x$

$= -\infty \times +\infty = -\infty$

$\Rightarrow \lim\limits_{x \to +\infty} f(x) = -\infty$

• $\lim\limits_{x \to +\infty} \dfrac{f(x)}{x} = \lim\limits_{x \to +\infty} \dfrac{(3-x)e^x}{x}$

$= \lim\limits_{x \to +\infty} 3-x \times \dfrac{e^x}{x}$

$= -\infty \times +\infty = -\infty$

$\Rightarrow \lim\limits_{x \to +\infty} \dfrac{f(x)}{x} = -\infty \Rightarrow$

b.p de direction (oy).

b) $f(x) = (3-x)e^x$.

$f'(x) = ((3-x)e^x)'$

$f'(x) = (3-x)'e^x + (3-x)(e^x)'$

$f'(x) = -e^x + (3-x)e^x$.

$f'(x) = (2-x)e^x$

$f'(x) = 0 \Rightarrow 2-x = 0$

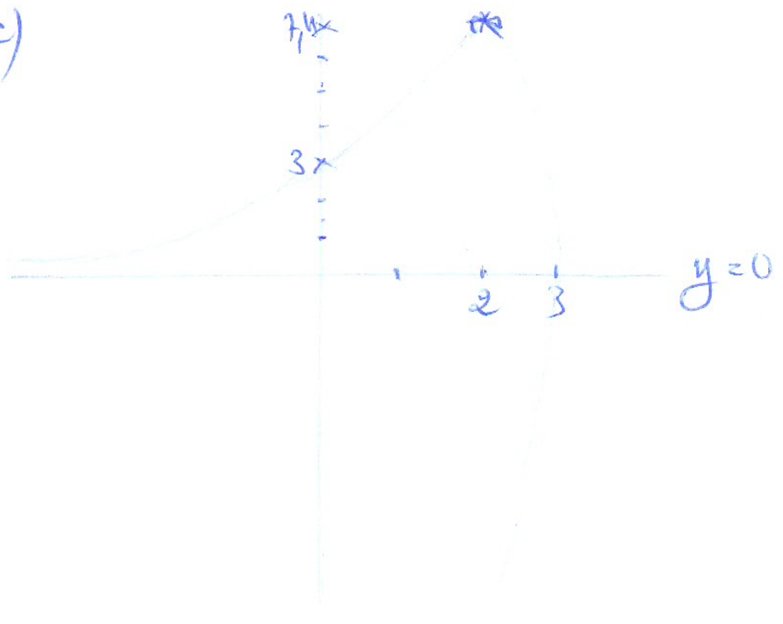
$\Rightarrow x = 2$

x	$-\infty$	2	$+\infty$
$f'(x)$	$+$		$-$
$f(x)$	$0 \nearrow$	$f(2) = 7{,}4$	$\searrow -\infty$

$\mathcal{C} \cap (ox)$: $f(x) = 0 \Rightarrow$ $(3, 0)$.

$\mathcal{C} \cap (oy)$: $f(0) = 3 \Rightarrow$ $(0, 3)$.

c)



d) : $y' - y = -e^x$?

$(2-x)e^x - (3-x)e^x =$

$(2-x-3+x)e^x = -e^x \Rightarrow$

$y' - y = -e^x$

$A = \int_0^3 f(x)\,dx$

$f'(x) - f(x) = -e^x$.

$f'(x) + e^x = f(x)$.

$\Rightarrow A = \int_0^3 f'(x) + e^x\,dx$.

$A = [f(x) + e^x]_0^3$

$A = (e^3 - 3 - 1)$

$A = e^3 - 4$.

2) $u_n = \frac{3^n}{n!}$

a) $n \geq 3$.

Mq: $0 \leq \frac{u_{n+1}}{u_n} \leq \frac{3}{4}$

$\frac{u_{n+1}}{u_n} = \frac{3^{n+1}/(n+1)!}{3^n/n!}$

$= \frac{3^{n+1} \times n!}{(n+1)! \times 3^n} = \frac{3^n \times 3 \times n!}{3^n \times n! \times (n+1)}$

$= \frac{3}{n+1}$.

$n \geq 3$

$3 \leq n \leq 0$

$4 \leq n+1 \leq 0$

$0 \leq \frac{1}{n+1} \leq \frac{1}{4} \Rightarrow$

$0 \leq \frac{3}{n+1} \leq \frac{3}{4}$.

b) Mq : $0 \leq u_n \leq \frac{9}{2}\left(\frac{3}{4}\right)^{n-3}$

$0 \leq \frac{\cancel{u_4}}{u_3} \leq \frac{3}{4}$

$0 \leq \frac{\cancel{u_5}}{\cancel{u_4}} \leq \frac{3}{4}$

$\vdots$

$0 \leq \frac{u_n}{\cancel{u_{n-1}}} \leq \frac{3}{4}$

$0 \leq \frac{u_n}{u_3} \leq \left(\frac{3}{4}\right)^{n-3}$

• $u_3 = \frac{3^3}{3!} = \frac{3 \times 9}{3 \times 2} = \frac{9}{2}$

$0 \leq \frac{u_n}{9/2} \leq \left(\frac{3}{4}\right)^{n-3}$

$0 \leq u_n \leq \frac{9}{2}(3/4)^{n-3}$

• $\lim\limits_{n \to +\infty} u_n = \lim\limits_{n \to +\infty} \frac{9}{2}(3/4)^{n-4}$

$= \frac{9}{2} \times 0$ car $\lim\limits_{n \to +\infty} \frac{3}{4}^{n-3} = 0$

$0 < \frac{3}{4} < 1$

$\Rightarrow \boxed{\lim\limits_{n \to +\infty} u_n = 0}$

3) $n \geq 1$.

$I_n = \frac{1}{n!} \int_0^3 (3-x)^n e^x dx$

$S_n = \sum_{k=0}^{n} \frac{3^k}{k!} = 1 + 3 + \frac{3^2}{2!} + \frac{3^3}{3!} + \dots + \frac{3^n}{n!}$

a) $I_1 = \int_0^3 (3-x) e^x dx$

$\Rightarrow I_1 = A = e^3 - 4$

b) Mq: $0 \leq I_n \leq (e^3-1) u_n$

$0 < x < 3$

$-3 < -x < 0$

$0 < 3-x < 3$

$0 < (3-x)^n < 3^n$

$0 < (3-x)^n e^x < 3^n e^x$

$0 < \int_0^3 (3-x)^n e^x dx < \int_0^3 3^n e^x dx$

$0 < \int_0^3 (3-x)^n e^x dx < 3^n [e^x]_0^3$

$0 < \int_0^3 (3-x)^n e^x dx < (e^3-1) 3^n$

$0 < \frac{1}{n!} \int_0^3 (3-x)^n e^x dx < (e^3-1) \frac{3^n}{n!}$

$\Rightarrow$

$0 < I_n < (e^3-1) u_n$.

• $\lim_{n \to +\infty} I_n = \lim_{n \to +\infty} (e^3-1) u_n$

$= (e^3-1) \times 0 = 0$

car $\lim_{n \to +\infty} u_n = 0$

$\Rightarrow \boxed{\lim_{n \to +\infty} I_n = 0}$

c) Mq: $I_{n+1} = I_n - u_{n+1}$.

$I_{n+1} = \frac{1}{n+1!} \int_0^3 (3-x)^{n+1} e^x dx$

$\begin{cases} u'(x) = e^x \\ V(x) = (3-x)^{n+1}. \end{cases}$

$I_{n+1} = \frac{1}{n+1!} \left[[e^x (3-x)^{n+1}]_0^3 + \int_0^3 e^x (n+1)(3-x)^n dx \right]$

$= \frac{1}{n+1!} \left(e^3 (3-3)^{n+1} - e^0 (3-0)^{n+1} + (n+1) \int_0^3 e^x (3-x)^n dx \right)$

$= \frac{-1}{n+1!} \times 3^{n+1} + \frac{1}{n+1!} (n+1) \times \int_0^3 e^x (3-x) dx$

$= -\frac{3^{n+1}}{n+1!} + \frac{1}{n!\cancel{(n+1)}} \cancel{(n+1)} \times \int_0^3 e^x (3-x) dx$

$\Rightarrow \boxed{I_{n+1} = I_n - u_{n+1}}$.

d) pour $n=1$: $\frac{3^0}{0!} + \frac{3^1}{1!} + I_1$

$= 1 + 3 + e^3 - 4 = e^3$

• On suppose que : $e^3 = 1 + 3 + \frac{3^2}{2!} + \dots +$

pour $n+1$: $1 + 3 + \frac{3^2}{2!} + \frac{3^3}{3!} + \dots + \frac{3^n}{n!} + \frac{3^{n+1}}{n+1!}$

$I_{n+1} = e^3 - I_n + u_{n+1} + I_n - u_{n+1}$

$= e^3 \Rightarrow e^3 = S_n + I_n$

$\Rightarrow S_n = e^3 - I_n \Rightarrow \lim_{n \to +\infty} S_n = \lim_{n \to +\infty} e^3 - \overset{0}{I_n} = e^3$.

Exercice 4

$$\begin{cases} g(x) = 1+x^3-3x^3\ln x \\ g(0)=1 \end{cases}$$

a) • $\lim\limits_{x\to 0^+} g(x) = \lim\limits_{x\to 0^+} 1+x^3-3x^3\ln x$

$= 1+0-3\times 0 \Rightarrow$

$\lim\limits_{x\to 0^+} g(x) = g(0) \Rightarrow$

g est continue en 0^+.

• $\lim\limits_{x\to +\infty} g(x) = \lim\limits_{x\to +\infty} 1+x^3-3x^2\ln x$

$= \lim\limits_{x\to +\infty} x^3\left(\frac{1}{x^3}+1-3\ln x\right)$

$= +\infty\,(0+1-3\times+\infty)$

$= +\infty \times -\infty = -\infty.$

• $\lim\limits_{x\to +\infty} g(x) = -\infty.$

b) $g'(x) = (1+x^3-3x^3\ln x)'$

$g'(x) = 3x^2-3\left((x^3)'\ln x + x^3(\ln x)'\right)$

$g'(x) = 3x^2-3\left(3x^2\ln x + \frac{x^3}{x}\right)$

$g'(x) = \cancel{3x^2}-9x^2\ln x-\cancel{3x^2}$

$g'(x) = -9x^2\ln x$

x	0		1		$+\infty$
$g'(x)$		$+$	0	$-$	
$g(x)$	1	$\nearrow$	2	$\searrow$	$-\infty$

c) $g(\alpha)=0$, g est décroissant sur $[1,+\infty[$.

$$\begin{cases} g(1)=2 \\ g(2)=-7{,}6 \end{cases}$$

$-7{,}6 < 0 < 2$

$g(2) < g(\alpha) < g(1)$

$\Rightarrow 1 < \alpha < 2.$

$g(x) = 1+(1-3\ln x)x^3$

$g(\alpha)=0 \Rightarrow$

x	0		α		$+\infty$
$g(x)$		$+$	0	$-$	

2) 1) $D_f =]0,+\infty[$, $f(x) = \frac{\ln x}{1+x^3}$

a) • $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0} \frac{\ln x}{1+x^3}$

$= \lim\limits_{x\to 0^+} \frac{1}{1+x^3}\times\ln x$

$= +\infty \times -\infty = -\infty$

$\Rightarrow \boxed{\lim\limits_{x\to 0^+} f(x) = -\infty}$

• $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} \frac{\ln x}{1+x^3} = \lim\limits_{x\to +\infty} \frac{x}{1+x^3}\times\frac{\ln x}{x}$

$= \lim\limits_{x\to +\infty} \frac{x}{x(\frac{1}{x}+x^2)} \times \frac{\ln x}{x} = 0\times 0 = 0$

$\Rightarrow \boxed{\lim\limits_{x\to +\infty} f(x) = 0}$

b) - $f'(x) = \left(\frac{\ln x}{1+x^3}\right)'$

$$f'(x) = \frac{(\ln x)'(1+x^3) - (\ln x)(1+x^3)'}{(1+x^3)^2}$$

$$f'(x) = \frac{1 + x^3/x - 3x^2 \ln x}{(1+x^3)^2}$$

$$f'(x) = \frac{1+x^3 - 3x^3 \ln x}{x} \times \frac{1}{(1+x^3)^2}$$

$$f'(x) = \frac{1+x^3 - 3x^3 \ln x}{x(1+x^3)^2}$$

$$f'(x) = \frac{g(x)}{x(1+x^3)^2}.$$

$x \in]0,+\infty[\Rightarrow x > 0$ et $(1+x^3)^2 > 0 \Rightarrow f'(x)$ a le même signe que $g(x)$.

x	0		α		$+\infty$
$f'(x)$		$+$	0	$-$	
$f(x)$	$-\infty$	$\nearrow$	$f(\alpha)$	$\searrow$	0

3) $x \in]1,+\infty[$

$F(x) = \int_1^x f(t)\,dt.$

a) f est continue sur $]0,+\infty[\Rightarrow f$ admet une primitive F dérivable sur $]1,+\infty[$.

$F'(x) = x' f(x) - (1)' f(1)$

$\Rightarrow F'(x) = f(x)$

$f(x) > 0 \Rightarrow F'(x) > 0 \Rightarrow$ F est croissante.

x	1		$+\infty$
$F'(x)$		$+$	
$F(x)$	0	$\nearrow$	$+\infty$

b) $t \in [1,+\infty[$.

$(1+t)^3 = 1^3 + 3t^2 + 3t + t^3 \geq t^3$

$\Rightarrow (1+t)^3 \geq t^3 \Rightarrow t^3 \leq (1+t)^3$.

$t^3 \leq 1+t^3 \leq (1+t)^3$

$$\frac{1}{(1+t)^3} \leq \frac{1}{1+t^3} \leq \frac{1}{t^3}$$

$t > 1 \Rightarrow \ln t > 0$

$\Rightarrow$

$$\frac{\ln t}{(1+t)^3} \leq \frac{\ln t}{1+t^3} \leq \frac{\ln t}{t^3}$$

$\Rightarrow$

$$\frac{\ln t}{1+t^3} \leq f(t) \leq \frac{\ln t}{t^3}.$$

$\Rightarrow \int_1^x \frac{\ln t}{t^3}\,dt$

$\begin{cases} u'(x) = 1/t^3 \\ v(x) = \ln t \end{cases}$

$\Rightarrow \int_1^x \frac{\ln t}{t^3}\,dt = 0$

$\left[-\frac{\ln t}{2t^2}\right]_1^x - \int_1^x -\frac{1}{2t^3}\,dt$

$= \left[-\frac{\ln t}{2t^2}\right]_1^x - \int_1^x -\frac{1}{2t^3}\,dt$

$= \left[-\frac{\ln t}{2t^2}\right]_1^x + \frac{1}{2}\int_1^x \frac{1}{t^3}\,dt$

$= \left[-\frac{\ln t}{2t^2}\right]_1^x + \frac{1}{2}\left[-\frac{1}{2t^2}\right]_1^x$

$= -\left[\frac{\ln t}{2t^2}\right]_1^x - \frac{1}{2}\left[\frac{1}{2t^2}\right]_1^x$

$= -\left(\frac{\ln x}{2x^2} - \frac{\ln 1}{2}^{0}\right)$

$-\frac{1}{2}\left(\frac{1}{2x^2} - \frac{1}{2}\right).$

$= \frac{-\ln x}{2x^2} - \frac{1}{4x^2} + \frac{1}{4}.$

d). $\frac{a}{t} + \frac{b}{1+t} + \frac{c}{(1+t)^2} =$

$\frac{a(1+t)(1+t)^2 + b(t)(1+t)^2 + c(t)(1+t)}{t(1+t)(1+t)^2}$

$= \frac{\cancel{(1+t)}\left(a(1+t)^2 + b(t)(1+t) + c(t)\right)}{t\cancel{(1+t)}(1+t)^2}$

$= \frac{a(1+t)^2 + bt(1+t) + ct}{t(1+t)^2}$

par identification:

$a(1+t)^2 + bt(1+t) + ct = 1$

$\Rightarrow 1 = a + 2at + at^2 + bt + bt^2 + ct$

$\Rightarrow 1 = (a+b)t^2 + (2a+b+c)t + a$

$\Rightarrow \begin{cases} a+b=0 \\ 2a+b+c=0 \\ a=1 \end{cases} \Rightarrow \begin{cases} 1+b=0 \\ 2+b+c=0 \\ a=1 \end{cases}$

$\Rightarrow \begin{cases} b=-1 \\ 2-1+c=0 \\ a=1 \end{cases} \Rightarrow \begin{cases} b=-1 \\ c=-1 \\ a=1 \end{cases}$

$\Rightarrow \frac{1}{t(1+t)^2} = \frac{1}{t} - \frac{1}{1+t} - \frac{1}{(1+t)^2}$

4) a) $\frac{\ln t}{(1+t)^3} \leq f(t) \leq \frac{\ln t}{t^3}$

$\Rightarrow \int_1^x \frac{\ln t}{(1+t)^3}\,dt \leq \int_1^x f(t)\,dt \leq \int_1^x \frac{\ln t}{t^3}\,dt.$

• Calculons: $\int_1^x \frac{\ln t}{(1+t)^3}\,dt$

$\begin{cases} u'(x) = 1/(1+t)^3 \\ v(x) = \ln t \end{cases} \Rightarrow$

$\int_1^x \frac{\ln t}{(1+t)^3}\,dt = \left[-\frac{\ln t}{2(1+t)^2}\right]_1^x$

$-\int_1^x \frac{-1}{2t(1+t)^2}\,dt$

$\Rightarrow$

$$\int_1^x \frac{\ln t}{(1+t)^3}\,dt = \left[\frac{-\ln t}{2(1+t)^2}\right]_1^x + \frac{1}{2}\int_1^x \frac{1}{t} - \frac{1}{1+t} - \frac{1}{(1+t)^2}\,dt$$

$$= -\left[\frac{\ln t}{2(1+t)^2}\right]_1^x + \frac{1}{2}\left[\ln t - \ln(1+t) + \frac{1}{1+t}\right]_1^x$$

$$= -\left(\frac{\ln x}{2(1+x)^2} - \frac{\ln 1}{8}\right) + \frac{1}{2}\left(\ln x - \ln(1+x) + \frac{1}{1+x} - \ln 1 + \ln 2 - \frac{1}{2}\right)$$

$$= \frac{-\ln x}{2(1+x)^2} + \frac{1}{2}\ln x - \frac{1}{2}\ln(1+x) + \frac{1}{2(1+x)} + \frac{1}{2}\ln 2 - \frac{1}{4}$$

$$= \frac{-\ln x}{2(1+x)^2} - \frac{1}{2}\left(\ln(1+x) - \ln x\right) + \frac{2\ln 2 - 1}{4} + \frac{1}{2(1+x)}$$

$$= \frac{-\ln x}{2(1+x)^2} - \frac{1}{2}\ln\left(\frac{x+1}{x}\right) - \frac{1-2\ln 2}{4} + \frac{1}{2(1+x)}$$

$\Rightarrow$ En remplaçant :

$$\frac{-\ln x}{2(1+x)^2} - \frac{1}{2}\ln\left(\frac{x+1}{x}\right) + \frac{1}{2(1+x)} - \frac{1-2\ln 2}{4} \leq F(x) \leq \frac{1}{4} - \frac{\ln x}{2x^2} - \frac{1}{4x^2}.$$

b) $\lim\limits_{x \to +\infty} F(x) = \ell$

$$\lim_{x\to+\infty} \frac{-\ln x}{2(1+x)^2} - \frac{1}{2}\ln\left(\frac{x+1}{x}\right) + \frac{1}{2(1+x)} - \frac{1-2\ln 2}{4} \leq \lim_{x\to+\infty} F(x) \leq \lim_{x\to+\infty} \frac{1}{4} - \frac{\ln x}{2x^2} - \frac{1}{4x^2}$$

- $\lim\limits_{x\to+\infty} \frac{-\ln x}{2(1+x)^2} - \frac{1}{2}\ln\left(\frac{x+1}{x}\right) + \cancel{\frac{1}{2(1+x)}}^{\,0} - \frac{1-2\ln 2}{4} = ?$

- $\lim\limits_{x\to+\infty} \frac{-\ln x}{2(1+x)^2} = \lim -\frac{1}{2} \times \frac{\ln x}{x} \times \frac{x}{(1+x)^2}$

$$= \lim_{x\to+\infty} -\frac{1}{2} \times \frac{\ln x}{x} \times \frac{\cancel{x}}{\cancel{x}\left(\frac{1}{x}+2+x\right)}$$

$$= -\frac{1}{2} \times 0 \times 0 = 0$$

- $\lim\limits_{x\to+\infty} -\frac{1}{2}\ln\left(\frac{x+1}{x}\right) = \lim\limits_{x\to+\infty} -\frac{1}{2}\ln\left(1+\frac{1}{x}\right)$

$$= \lim_{x\to+\infty} -\frac{1}{2x} \times x\ln\left(1+\frac{1}{x}\right)$$

$$= 0 \times 1 = 0.$$

$\Longrightarrow$

$$\lim_{x\to+\infty} -\frac{\ln x}{2(1+x)^2}{}^{0} - \frac{1}{2}\ln\left(\frac{x+1}{x}\right) + \frac{1}{2(x+1)}{}^{0} - \frac{1-2\ln 2}{4}$$

$$= -\frac{1-2\ln 2}{4} = -\frac{1}{4} + \frac{\ln 2}{2}$$

- $\lim\limits_{x\to+\infty} \frac{1}{4} - \frac{1}{4x^2} - \frac{\ln x}{2x^2}$

$$= \frac{1}{4} - \lim_{x\to+\infty} \frac{\ln x}{2x^2}$$

- $\lim\limits_{x\to+\infty} \frac{\ln x}{2x^2} = \lim\limits_{x\to+\infty} \frac{1}{2x} \times \frac{\ln x}{x}$

$$= 0 \times 0 = 0$$

$$\Longrightarrow \lim_{x\to+\infty} \frac{1}{4} - \frac{1}{4x^2} - \frac{\ln x}{2x^2} = \frac{1}{4}$$

$\Longrightarrow$ Par suite :

$$-\frac{1}{4} + \frac{1}{2}\ln(2) \leq \ell \leq \frac{1}{4}.$$

c)