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EXERCICES

Exercice 1

Mettre sous la forme algébrique les nombres :

$$z_1 = (2+5i)(3+2i),\ z_2 = \frac{2+3i}{3+2i}, z_3 = \frac{2i+3}{5-i} + 2 - 7i,\ z_4 = (1-2i)^3,$$

$$z_5 = (3+i)(2+3i)(1-2i),\ z_6 = \frac{4-5i}{2+3i} + \frac{2-3i}{5+2i} + 1 - 2i.$$

Réponse

- Le Module et argement de nombre complexe

- $|z_1| = |2-2i| = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$

$\boxed{|z_1| = 2\sqrt{2}}$

- Soit $\theta_1 = \arg z_1 \Rightarrow \begin{cases} \cos\theta_1 = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \\ \sin\theta_1 = \frac{-2}{2\sqrt{2}} = \frac{-1}{\sqrt{2}} \end{cases} \Rightarrow \boxed{\theta_1 = \frac{-\pi}{4}}$

- $|z_2| = |-\sqrt{3}+i| = \sqrt{3+1} = \sqrt{4} = 2$

$|z_2| = 2$

- Soit $\theta_2 = \arg z_2 \Rightarrow \begin{cases} \cos\theta_2 = \frac{-\sqrt{3}}{2} \\ \sin\theta_2 = \frac{1}{2} \end{cases}$

$\Rightarrow \theta_2 = \pi - \frac{\pi}{6} \Rightarrow \boxed{\arg z_2 = \frac{5\pi}{6}}$

$\bullet \left|z_3\right| = \left|\frac{\sqrt{2+2i}}{\sqrt{3}-i}\right| = \frac{\sqrt{2^2+2^2}}{\sqrt{3+1}} = \frac{2\sqrt{2}}{2} \Rightarrow \boxed{z_3 = \frac{2\sqrt{2}}{2}}$

$\bullet$ Soit $z_3 = \frac{2+2i}{\sqrt{3}+i} \Rightarrow \frac{2\sqrt{2}\, e^{i\frac{\pi}{4}}}{2\, e^{-i\frac{\pi}{6}}} = \sqrt{2}\, e^{i\left(\frac{\pi}{4}+\frac{\pi}{6}\right)} = \frac{10\pi}{24}$

$\Rightarrow \boxed{\arg z_3 = \frac{10\pi}{24}}$

$\bullet\ z_4 = (3+3i)^5 \cdot (-3+3i)^2 \Rightarrow \left|z_4\right| = \left|3+3i\right|^5 \cdot \left|-3+3i\right|^2$

$= (3\sqrt{2})^5 \times (3\sqrt{2})^2 \Rightarrow \boxed{\left|z_4\right| = (3\sqrt{2})^7}$

$\arg z_4 = \arg (3+3i)^5 - \arg(-3+3i)^2$

$= 5\arg(3+3i) + 2\arg(-3+3i) = 5\left(\frac{\pi}{4}\right) + 2\left(\frac{3\pi}{4}\right)$

$\Rightarrow \arg z_4 = \frac{11\pi}{4} = 2\pi + \frac{3\pi}{4} = \boxed{\frac{3\pi}{4}\ [2\pi]}$

$\bullet \left|z_5\right| = \frac{\left|2+2i\right|^7 \cdot \left|\sqrt{3}-i\right|^2}{\left|1+i\sqrt{3}\right|^8} = \frac{(2\sqrt{2})^7 \cdot 2^2}{2^8} = 2^7 \times \sqrt{2}^7 \times 2^2 \times 2^{-8}$

$= 2(\sqrt{2})^6 \times (\sqrt{2})^1 = 2 \times 2^3 \times \sqrt{2} \Rightarrow \boxed{\left|z_5\right| = 2^4\sqrt{2}}$

$\bullet \arg z_5 = \arg(2+2i)^7 + \arg(\sqrt{3}-i)^2 - \arg(1+i\sqrt{3})^8$

$= 7\arg(2+2i) + 2\arg(\sqrt{3}-i) - 8\arg(1+i\sqrt{3})$

$= 7\left(\frac{\pi}{4}\right) + 2\left(\frac{-\pi}{6}\right) - 8\left(\frac{\pi}{3}\right)$

$\arg z_5 = \frac{7\pi}{4} - \frac{\pi}{3} - \frac{8\pi}{3} = \frac{7\pi}{4} - \pi \Rightarrow \boxed{\arg z_5 = \frac{3\pi}{4}\ [2\pi]}$

$\bullet\ z_6 = \dfrac{2i}{(-2\sqrt{3}+2i)^6} \Rightarrow |z_6| = \dfrac{|2i|}{|-2\sqrt{3}+2i|^6}$

$|z_6| = 2 \times 4^6 = 2(2^2)^{-6} = 2\times(2)^{-12} = 2^{-11}$

$$\boxed{|z_6| = 2^{-11}}$$

$\bullet\ \arg z_6 = \arg(2i) - \arg(-2\sqrt{3}+2i)^6$

$\arg z_6 = \arg(2i) - 6\arg(-2\sqrt{3}+2i)$.

$= \dfrac{\pi}{2} - 6\left(\dfrac{5\pi}{6}\right)$.

$\arg z_6 = \dfrac{\pi}{2} - 5\pi = \dfrac{\pi}{2} - \pi = -\dfrac{\pi}{2}\ [2\pi]$.

$$\boxed{\arg z_6 = -\frac{\pi}{2}\ [2\pi]}.$$