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Exercice de nombres complexes

Exercice

Résoudre dans  $\mathbb{C}$  l'équation suivante :

$$10z^4 - (39-3i)z^3 + 60z^2 - (39-3i)z + 10 = 0$$

(On pose  $Z = z + \frac{1}{z}$ ).

Solution :

(E):  $10z^4 - (39-3i)z^3 + 60z^2 - (39-3i)z + 10 = 0$

On constate que  $z=0$  n'est pas une solution de (E).

Divisons par  $z^2$ :

$$10z^2 - (39-3i)z + 60 - (39-3i)\frac{1+10z^2}{z^2} = 0$$

$$\Rightarrow 10(z^2 + \frac{1}{z^2}) - (39-3i)(z + \frac{1}{z}) + 60 = 0$$

$$\Rightarrow 10(z^2 + \frac{1}{z^2} + 2 - 2) - (39-3i)(z + \frac{1}{z}) + 60 = 0$$

Si  $Z = z + \frac{1}{z}$ , alors  $Z^2 = (z + \frac{1}{z})^2 = z^2 + 2 + \frac{1}{z^2}$

et on a alors:

$$\Rightarrow 10(Z^2 - 2) - (39-3i)Z + 60 = 0$$

$$\Rightarrow 10Z^2 - (39-3i)Z + 40 = 0$$

$$\Delta = (39-3i)^2 - 4 \times 10 \times 40 = -88 - 234i = (9-13i)^2$$

$$\boxed{\zeta = 9-13i}$$

$$* Z_1 = \frac{39-3i+9-13i}{20} = \frac{48-16i}{20}$$

$$Z_1 = \frac{12-4i}{5}$$

$$* Z_2 = \frac{39-3i-9+13i}{20} = \frac{30+10i}{20}$$

$$Z_2 = \frac{3+i}{2}$$

Si  $Z = Z_1 \Rightarrow z + \frac{1}{z} = \frac{12-4i}{5}$

$$\Rightarrow \frac{z^2+1}{z} = \frac{12-4i}{5}$$

$$\boxed{5z^2 - (12-4i)z + 5 = 0} \Rightarrow \Delta = 28-96i$$

$$\Rightarrow \Delta = (8-6i)^2 \Rightarrow \zeta = (8-6i)$$

$$z_1 = \frac{12-4i+8-6i}{10} = \frac{20-10i}{10} = 2-i$$

$$z_2 = \frac{12-4i-8+6i}{10} = \frac{4+2i}{10} = \frac{2+i}{5}$$

Si  $Z = Z_2 \Rightarrow z + \frac{1}{z} = \frac{3+i}{2} \Rightarrow \frac{z^2+1}{z} = \frac{3+i}{2}$

$$\boxed{2z^2 - (3+i)z + 2 = 0} \Rightarrow \Delta = -8+6i$$

$$\Rightarrow \Delta = (1+3i)^2 \Rightarrow \zeta = (1+3i)$$

$$z_3 = \frac{3+i+1+3i}{4} = 1+i$$

$$z_4 = \frac{3+i-1-3i}{4} = \frac{1-i}{2}$$

$$S = \left\{ 2-i; \frac{2+i}{5}; 1+i; \frac{1-i}{2} \right\}$$

$5z^2 - (12-4i)z + 5 = 0$   
 $\Delta = 28-96i = (8-6i)^2$