Gassoum Chouaibou Ware

Exercice 10 :

En utilisant le changement de variable, calculer les intégrals:

$I_1 = \int_1^2 \frac{dx}{(4x+5)^5}$; $t = 4x+5$ $\qquad$ $I_2 = \int_1^{\sqrt{3}} \frac{dx}{1+x^2}$; $x = \tan t$

$I_3 = \int_0^{\frac{\pi}{2}} \frac{4dx}{1+\cos x}$; $t = \tan\frac{x}{2}$ $\qquad$ $I_4 = \int_2^3 \frac{x^3dx}{\sqrt{x-1}}$; $t = x-1$

$I_5 = \int_0^2 \frac{\sqrt{x+1}}{x^2+3x+2}dx$; $t = \sqrt{1+x}$ puis $t = \tan u$

Solution

* Methode:

1. Changement de bornes
2. Relation entre différentiells
3. Remplacement
4. Calcul

- $I_1 = \int_1^2 \frac{dx}{(4x+5)^5}$ $\quad t = 4x+5$ $\quad \begin{cases} x=1 \Rightarrow t=9 \\ x=2 \Rightarrow t=13 \end{cases}$

$dt = 4dx \Rightarrow dx = \frac{1}{4}dt$

$I_1 = \int_9^{13} \frac{1}{4} \times \frac{1}{t^5}dt = \frac{1}{4}\int_9^{13} t^{-5}dt = \frac{1}{4}\left[-\frac{1}{4}t^{-4}\right]_9^{13} = \frac{-1}{16}(13^{-4} - 9^{-4})$

D'ou $\boxed{I_1 = \frac{-1}{16}(13^{-4} - 9^{-4})}$

- $I_2 = \int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ $\quad x = \tan t$ $\quad \begin{cases} x=1 \Rightarrow \tan t = 1 \Rightarrow t = \frac{\pi}{4} \\ x=\sqrt{3} \Rightarrow \tan t = \sqrt{3} \Rightarrow t = \frac{\pi}{3} \end{cases}$

$dx = (1+\tan^2 t)dt \Rightarrow I_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{(1+\tan^2 t)dt}{(1+\tan^2 t)} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} dt$

$I_2 = [t]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12} \Rightarrow \boxed{I_2 = \frac{\pi}{12}}$

$-I_3 = \int_0^{\frac{\pi}{2}} \frac{4dx}{1+\cos x}$, $t = \tan \frac{x}{2}$ $\quad \begin{vmatrix} x=0 \Rightarrow t = \tan 0 = 0 \\ x=\frac{\pi}{2} \Rightarrow t = \tan\frac{\pi}{4} = 1 \end{vmatrix}$

$dt = \frac{1}{2}\left(1+\tan^2\frac{x}{2}\right)dx \Rightarrow dt = \frac{1}{2\cos^2\frac{x}{2}}dx$

$dx = 2\cos^2\frac{x}{2}\,dt \Rightarrow I_3 = \int_0^1 \frac{4\times 2\cos^2\frac{x}{2}\,dt}{2\cos^2\frac{x}{2}}$

Car $1+\cos\alpha = 2\cos^2\frac{\alpha}{2}$ Donc $I_3 = \int_0^1 4\frac{t}{t}dt = \int_0^1 4\,dt$

$I_3 = [4t]_0^1 = 4 - 4\times 0 = 4$ Donc $\boxed{I_3 = 4}$

$-I_4 = \int_2^3 \frac{x^3dx}{\sqrt{x-1}}$, $t = x-1$ $\quad \begin{vmatrix} x=2 \Rightarrow t=1 \\ x=3 \Rightarrow t=2 \end{vmatrix}$

$dt = dx$ d'abord $\frac{x^3}{\sqrt{x-1}} = \frac{(t+1)^3}{\sqrt{t}} = \frac{t^3+3t^2+3t+1}{\sqrt{t}}$

on a Donc $\frac{x^3}{\sqrt{x-1}} = \frac{t^3}{\sqrt{t}} + \frac{3t^2}{\sqrt{t}} + \frac{3t}{\sqrt{t}} + \frac{1}{\sqrt{t}} = t^{\frac{5}{2}} + 3t^{\frac{3}{2}} + 3t^{\frac{1}{2}} + t^{-\frac{1}{2}}$

D'où $I_4 = \left[\frac{1}{\frac{5}{2}+1}t^{\frac{5}{2}+1} + \frac{3}{\frac{3}{2}+1}t^{\frac{3}{2}+1} + \frac{3}{\frac{1}{2}+1}t^{\frac{1}{2}+1} + \frac{1}{-\frac{1}{2}+1}t^{-\frac{1}{2}+1}\right]_1^2$

$\Rightarrow I_4 = \left[\frac{2}{7}t^{\frac{7}{2}} + \frac{6}{5}t^{\frac{5}{2}} + 2t^{\frac{3}{2}} + 2t^{\frac{1}{2}}\right]_1^2$

$\Rightarrow I_4 = \left[\left(\frac{2}{7}t^3 + \frac{6}{5}t^2 + 2t + 2\right)\sqrt{t}\right]_1^2 = \left(\frac{16}{7} + \frac{24}{5} + 6\right)\sqrt{2} - \left(\frac{2}{7} + \frac{6}{5} + 4\right)$

par suite $\boxed{I_4 = \frac{458}{35}\sqrt{2} - \frac{192}{35}}$

$- I_5 = \int_0^2 \frac{\sqrt{x+1}}{x^2+3x+1}\,dx$, $t = \sqrt{x+1}$: 1er Étape

$\Rightarrow \left| \begin{array}{l} x=0 \Rightarrow t=1 \\ x=2 \Rightarrow t=\sqrt{3} \end{array} \right.$ $dt = \frac{dx}{2\sqrt{x+1}} = \frac{dx}{2t} \Rightarrow dx = 2t\,dt$

Factorisons le dénominateur $x^2+3x+1 = (x+1)(x+2) = t^2(t^2+1)$

On a donc $I_5 = \int_1^{\sqrt{3}} \frac{t \times 2t}{t^2(t^2+1)}\,dt = 2\int_1^{\sqrt{3}} \frac{t^2}{t^2(t^2+1)}\,dt = 2\int_1^{\sqrt{3}} \frac{1}{t^2+1}\,dt$

2ème Étape : $t = \tan u$ $\left| \begin{array}{l} t=1 \Rightarrow \tan u = 1 \Rightarrow u = \frac{\pi}{4} \\ t=\sqrt{3} \Rightarrow \tan u = \sqrt{3} \Rightarrow u = \frac{\pi}{3} \end{array} \right.$

$dt = (1+\tan^2 u)\,du$

$\Downarrow$

$I_5 = 2\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{(1+\tan^2 u)}{(1+\tan^2 u)}\,du = 2\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} du = 2[u]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

Donc $\boxed{I_5 = \frac{\pi}{6}}$