ECOOLE PRIVE ERRAJA

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CORRECTION:
EXERCICE :4
BAC
2015 SC

1er f.p

Bac 2015 Sc :

Ex: 4

- Soit $f(x) = \frac{x+1}{e^x} = (x+1)e^{-x}$

1) a) Dressons le Tableau de Variation de f :

$\lim\limits_{-\infty} f(x) = -\infty \cdot +\infty = -\infty$

$\lim\limits_{+\infty} f(x) = \lim\limits_{+\infty} (x+1)e^{-x} = \lim\limits_{+\infty}\left(\frac{x}{e^x}+\frac{1}{e^x}\right)$

$= 0$

$f'(x) = e^{-x} - (x+1)e^{-x} = -xe^{-x}$

$f'(x) = 0 \Rightarrow -x = 0 \Rightarrow x = 0$

x	$-\infty$		0		$+\infty$
$f'(x)$		$+$	0	$-$	
f	$-\infty$	$\nearrow$	1	$\searrow$	0

1-b) Tracé de C_f

$C \cap ox$: $f(x) = 0 \Rightarrow x+1 = 0 \Rightarrow x = -1$

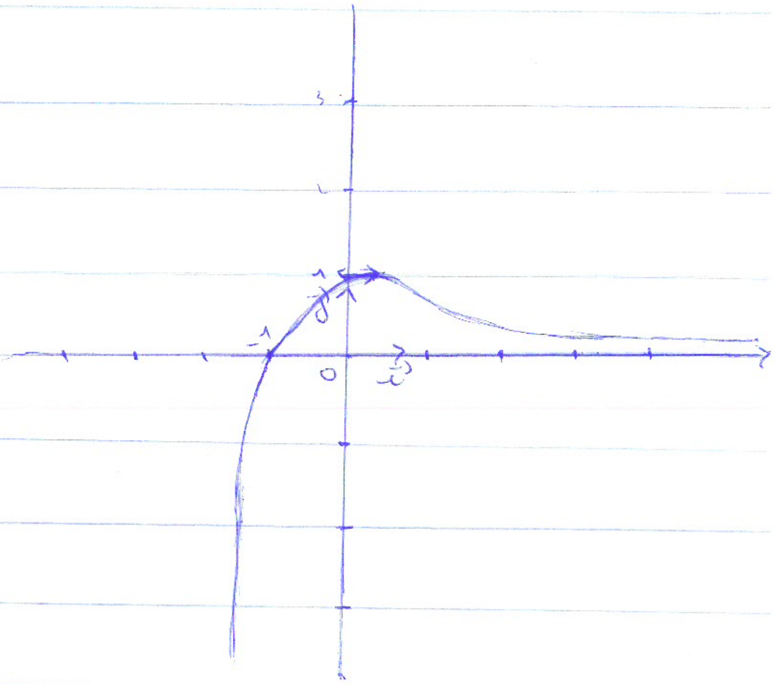
$A(-1, 0)$

$C \cap oy$: $f(0) = 1 \Rightarrow B(0, 1)$

$y = 0$ A.H au $V(+\infty)$

$\lim\limits_{-\infty} \frac{f(x)}{x} = \lim\limits_{-\infty}\left[\frac{x+1}{x} \cdot e^{-x}\right] = +\infty$

C admet B.p de direction (oy)



2°) $\forall n \geq 1$ $f_n(x) = \frac{(1+x)^n}{e^x} = (1+x)^n e^{-x}$

$\forall x \in \mathbb{R}$ $F_n(x) = \int$

Montrons que $\forall n \geq 1$ et $\forall x \in \mathbb{R}$

$F_{n+1}(x) = (n+1)F_n(x) - f_{n+1}(x)$

Par intégration par parties

$F_{n+1}(x) = \int_{-1}^{x} (1+x)^{n+1} e^{-x}\,dx$

on pose $\begin{cases} U(x) = (1+x)^{n+1} \longrightarrow U'(x) = (n+1)(1+x)^n \\ V'(x) = e^{-x} \longrightarrow V(x) = -e^{-x} \end{cases}$

$F_{n+1}(x) = \left[-(1+x)^{n+1} e^{-x}\right] + (n+1)\int_{-1}^{x} (1+t)^{n} e^{-t}\,dt$

$\cancel{F_{n+1}(x) = [-(1+x)^{n+1}}$

$F_{n+1}(x) = -f_{n+1}(x) + (n+1)F_n(x)$

3°) Soit : $I_n = F_n(0) = \int_{-1}^{0} f_n(t)\,dt$

a°) Vérifions que $\forall n \geq 1$

$I_{n+1} = (n+1)I_n - 1$

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Ex : 4 (Suite)

4°a) (Suite)

Par identification :

$U_n = U_1 - \left(\frac{1}{2!} + \frac{1}{3!} \cdots + \frac{1}{n!}\right)$

$U_n = e - 2 - \left(\frac{1}{2!} + \frac{1}{3!} \cdots + \frac{1}{n!}\right)$

$= e - \frac{1}{0!} - \frac{1}{1!} - \left(\frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}\right)$

$= e - \left(\frac{1}{0!} + \frac{1}{1!} \cdots + \frac{1}{n!}\right)$

$\boxed{U_n = e - \sum_{k=0}^{n} \frac{1}{k}!}$

b°) Calcul de $\lim_{+\infty} \sum_{k=1}^{n} \frac{1}{k}!$

on sait que :

$\lim_{+\infty} U_n = \lim_{+\infty} \frac{I_n}{n} = 0$

$\sum_{k=0}^{n} \frac{1}{k}! = e - U_n$

$\Rightarrow \boxed{\lim_{+\infty} \sum_{k=0}^{n} \frac{1}{k}! = e}$